Continuum theory course,
Gal Sarid,
Spring 2006

משוואות התנועה עבור זורם בלתי-דחיס

$\vec{u}=\left(u_r,u_\phi,u_z\right)$ - קואורדינטות גליליות (Cylindrical Polar coordinates)

משוואת הרציפות (Continuity eq.):

$$\frac{1}{r}\frac{\partial\left(ru_r\right)}{\partial r}+\frac{1}{r}\frac{\partial u_\phi}{\partial\phi}+\frac{\partial u_z}{\partial z}$$

משוואת Navier-Stokes:

$$\frac{\partial u_r}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)u_r-\frac{{u_\phi}^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\nabla^2u_r-\frac{2}{r^2}\frac{\partial u_\phi}{\partial\phi}-\frac{u_r}{r^2}\right),$$

$$\frac{\partial u_\phi}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)u_\phi-\frac{u_ru_\phi}{r}=-\frac{1}{\rho r}\frac{\partial p}{\partial\phi}+\nu\left(\nabla^2u_\phi-\frac{2}{r^2}\frac{\partial u_r}{\partial\phi}-\frac{u_\phi}{r^2}\right),$$

$$\frac{\partial u_z}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)u_z=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\nabla^2u_z,$$

טנזור המאמץ (Stress Tensor):

$$\tau_{rr}=-p+2\mu\frac{\partial u_r}{\partial r}\qquad\tau_{r\phi}=\mu\left(\frac{1}{r}\frac{\partial u_r}{\partial\phi}+\frac{\partial u_\phi}{\partial r}-\frac{u_\phi}{r}\right)$$

$$\tau_{\phi\phi}=-p+2\mu\left(\frac{1}{r}\frac{\partial u_\phi}{\partial\phi}+\frac{u_r}{r}\right)\qquad\tau_{\phi z}=\mu\left(\frac{\partial u_\phi}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial\phi}\right)$$

$$\tau_{zz}=-p+2\mu\frac{\partial u_z}{\partial z}\qquad\tau_{zr}=\mu\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)$$

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$u = (u_r, u_\theta, u_\phi)$ - (Spherical Polar coordinates) קואורדינטות כדוריות

:(Continuity eq.) משוואת הרציפות

$$\frac{1}{r^2}\frac{\partial(r^2 u_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(u_\theta \sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(u_\phi)}{\partial\phi}$$

:Navier-Stokes משוואת

$$\frac{\partial u_r}{\partial t} + (\vec{u}\cdot\vec{\nabla})u_r - \frac{u_\theta^{\,2} - u_\phi^{\,2}}{r}$$
$$= -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\nabla^2 u_r - \frac{2}{r^2\sin^2\theta}\frac{\partial(u_\theta\sin\theta)}{\partial\theta} - \frac{2}{r^2\sin\theta}\frac{\partial u_\phi}{\partial\phi} - \frac{2u_r}{r^2}\right),$$
$$\frac{\partial u_\theta}{\partial t} + (\vec{u}\cdot\vec{\nabla})u_\theta + \frac{u_r u_\theta}{r} - \frac{u_\phi^{\,2}\cot\theta}{r}$$
$$= -\frac{1}{\rho r}\frac{\partial p}{\partial\theta} + \nu\left(\nabla^2 u_\theta - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\phi}{\partial\phi} + \frac{2}{r^2}\frac{\partial u_r}{\partial\theta} - \frac{u_\theta}{r^2\sin^2\theta}\right),$$
$$\frac{\partial u_\phi}{\partial t} + (\vec{u}\cdot\vec{\nabla})u_\phi + \frac{u_r u_\phi}{r} + \frac{u_\theta u_\phi\cot\theta}{r}$$
$$= -\frac{1}{\rho r\sin\theta}\frac{\partial p}{\partial\phi} + \nu\left(\nabla^2 u_\phi + \frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\theta}{\partial\phi} - \frac{u_\phi}{r^2\sin^2\theta}\right),$$

:(Stress Tensor) טנזור המאמץ

$$\tau_{rr} = -p + 2\mu\frac{\partial u_r}{\partial r}$$
$$\tau_{\phi\phi} = -p + 2\mu\left(\frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial\phi} + \frac{u_r}{r} + \frac{u_\theta\cot\theta}{r}\right)$$
$$\tau_{\theta\theta} = -p + 2\mu\left(\frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_r}{r}\right)$$
$$\tau_{r\theta} = \mu\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)$$
$$\tau_{\theta\phi} = \mu\left(\frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial\phi} + \frac{1}{r}\frac{\partial u_\phi}{\partial\theta} - \frac{u_\phi\cot\theta}{r}\right)$$
$$\tau_{\phi r} = \mu\left(\frac{\partial u_\phi}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial u_r}{\partial\phi} - \frac{u_\phi}{r}\right)$$